Chapter 5

Investment adjustment costs

Introduction

In the standard DSGE model it is assumed that capital stock can be changed from one period to another without any restriction, through the investment process. Thus, given a particular shock affecting optimal capital stock, agents can change their investment decisions such that the resulting capital stock would be again the optimal without any transformation cost. However, in the real world, physical capital is a special variable, because of its particular characteristics. We are speaking about factories, machines, ships, etc., that cannot be built up instantaneously or need to be installed to produce. One important aspect to be considered here is that the investment process is subject to implicit costs which are missing in the basic theoretical setup. This will causes additional rigidities in the capital accumulation process. This means that in the case that the capital stock is not at the optimal level, agents do not take an investment decision to completely cover the difference in a single period of time, but they change capital stock in a gradual process over time as investment is smoothed.

In the literature, the above question have been studied using two alternative approaches: Considering the existence of adjustment costs in investment or, alternatively, by considering the existence of adjustment costs relative to the capital stock. In the first case, we face a cost associated with the variation in the level of investment compared to its steady state value. In the second case, we are talking about a cost in terms of the change in the capital stock. Both concepts are broadly equivalent, although they involve different specifications of the adjustment cost process. In this chapter, we will focus on the existence of adjustment costs associated with the investment process which are the more common adjustment cost considered in the literature. Investment decisions are costly in terms of loss of consumption given that a fraction of the output that goes to investment disappears, i.e., fails to be transformed into capital.

The structure of the rest of the chapter is as follows. Section 2 briefly reviews the concept of adjustment costs in investment and the different approaches used in the literature. Section 3 develops a DSGE model with adjustment costs in investment which are added to the capital accumulation equation. Section 4 presents the equations of the model and the calibration exercise. Section 5 studies the dynamic effects of a productivity shock. The
Investment adjustment costs

In the standard DSGE model the treatment given to the productive sector of the economy is very simple. Firms maximize profits period by period, by solving a static problem. In practice, firms take decisions on an intertemporal context, so the right thing would be to specify the problem in terms of maximizing the sum of all discounted profits. However, if we solve this dynamic problem, the result we get is exactly the same as in the static case, indicating that firms decisions today will not affect future profits, which does not seem to make much sense. This result occurs because the assumptions regarding the behavior of the firms are overly restrictive.

One of the shortcomings of the neoclassical analysis of the firm comes from the assumption that there is no restriction to the instantaneous variation in the capital stock and investment simply transforms into installed capital. However, in reality, firms face adjustment costs by altering their capital stock. The literature distinguishes between two types of adjustment costs: external and internal. External adjustment costs arise when firms face a perfectly elastic supply of capital. This will cause the price of capital to depend on the velocity of installation and/or on the quantity of new capital. By contrast, the internal adjustment costs are measured in terms of production losses. When new capital should be installed, a portion of the investment must be expended in the installation process which is no-costly or, alternatively, a fraction of the inputs already used in the production, basically labor, must be devoted to the installation of the new capital. These inputs will be not available to produce during the installation process, which implies forgone output.

Investment and capital accumulation analysis can take place either from the point of view of the firm or from the point of view of households, depending on the assumption about who is the owner of the capital stock. Strictly, the most realistic option appears to be the first, as it is firms that decide the level of investment in each period. This approach has been widely used to study the investment function, leading to the so-called Tobin’s Q theory (Tobin, 1969; Hayashi, 1982), which allows to study the investment process based on the dynamics of the Q ratio that represents the ratio between the market value of the firm and the replacement cost of its installed capital. The alternative option, which is commonly used in DSGE models, involves studying the investment adjustment costs from the point of view of households. This is simply because we assume that the households are the owners of the capital stock.
In general, we can distinguish between capital adjustment costs and investment adjustment costs. Jorgenson (1963) introduced the existence of adjustment costs of investment as a lag structure associated with the investment process. Tobin (1969) developed a theory in which the investment decisions are taken depending on the value of a ratio named Q, defined as the market value of the firm relative to the replacement cost of installed capital. Hayashi (1982) shows that under certain conditions this ratio is equal to its marginal, the so-called \( q \)-ratio.

The existence of capital adjustment costs has been considered extensively in the literature on investment by Hayashi (1982), Abel and Blanchard (1993), Shapiro (1986), among others. Generally, we can define the following function for capital adjustment costs:

\[
\Psi(\cdot) = \Psi(I_t / K_t)
\]  

where the adjustment cost function, \( \Psi(\cdot) \), depends on the quantity of investment, \( I_t \), relative to the installed capital stock, \( K_t \), that is, on the ratio between the new capital to be installed and the capital stock already installed. This cost function has a number of features, such that:

\[
\Psi(\delta) = 0 \\
\Psi'(I_t / K_t) > 0 \\
\Psi''(I_t / K_t) > 0
\]

i.e., adjustment costs depend positively on investment relative to capital stock. If net investment is zero, gross investment is just equal to capital loss due to depreciation. Furthermore, its second derivative is positive, indicating that adjustment costs is convex. The existence of adjustment costs means a capital loss or an additional cost in the investment process. So for each dollar invested, it will transform into capital an amount less than one dollar, as a consequence of the adjustment costs. In this setting, the marginal productivity of capital is also a function of net investment adjustment costs.

Alternatively, the adjustment costs associated with investment refer to the existence of costs in terms of investment changes between periods. The usual way to define the adjustment cost of investment function is as follows (see, for instance, Christiano, Eichenbaum and Evans, 2005):

\[
\Psi(\cdot) = \Psi\left(\frac{I_t}{I_{t-1}}\right)
\]  

(5.2)
where

$$
\Psi(1) = 0 \\
\Psi'(1) = 0 \\
\Psi''(1) > 0
$$

implying that there is a cost associated with changing the level of investment, that this cost is zero at steady state, and that this cost is increasing in the change in investment. Using this specification, the capital accumulation equation is defined as:

$$
K_{t+1} = (1 - \delta)K_t + \left[ 1 - \Psi\left( \frac{I_t}{I_{t-1}} \right) \right] I_t 
$$

(5.3)

In the literature we find a large number of DSGE models including the existence of adjustment costs either in capital or investment. Adjustment costs in capital have been considered, by Jermann (1998), Edge (2000), F-de-Córdoba and Kehoe (2000) and Boldrin, Christiano and Fisher (2001), among many others. For example, Edge (2000) shows that adjustment costs in capital together with habit persistence in consumption in a sticky-price monetary model is capable of generating a liquidity effect (a decline in short-term nominal interest rate in response to a positive monetary shock).

Adjustment costs in investment have been also considered extensively in the literature. For instance, Christiano et al. (2005) show that adjustment costs on investment can generate a hump-shaped response in investment, consumption and employment, consistent with the estimated response to a monetary policy shock. Finally, Burnside, Eichenbaum and Fisher (2004) show that an RBC model with adjustment costs in investment may explain the effects of a fiscal shock on hours worked and wages.

The model

The DSGE model presented here introduces the existence of adjustment costs in the investment process. This means that we will now alter the capital accumulation equation, including a cost function of investment adjustment. In this setting, consumers now must make a further decision as investment adjustment costs are incorporated in the budget constraint. This is because optimal capital stock decision and investment decision are now separated due to the existence of adjustment costs in the investment process.
Households

It is assumed that households maximize their intertemporal utility function in terms of consumption, \(\{C_t\}_{t=0}^{\infty}\), and leisure, \(\{1-L_t\}_{t=0}^{\infty}\), where \(L_t\) denotes labor. Consumer’s preferences are defined by the following utility function:

\[
\sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_t + (1-\gamma) \log(1-L_t) \right]
\]  
(5.4)

where \(\beta\) is the discount factor and where \(\gamma \in (0, 1)\) is the proportion of consumption on total income.

Consumer’s budget constraint states that consumption plus saving, \(S_t\), cannot exceed the sum of labor and capital rental income:

\[
C_t + S_t = W_t L_t + R_t K_t
\]

where \(W_t\) is the wage, \(R_t\) is the rental price of capital and \(K_t\) is the physical capital stock. Investment adjustment costs are introduced by assuming the following equation for capital accumulation:

\[
K_{t+1} = (1-\delta) K_t + \left[ 1 - \Psi \left( \frac{I_t}{I_{t-1}} \right) \right] I_t
\]

(5.5)

where \(\delta\) is the physical capital depreciation rate, \(I_t\) is gross investment and \(\Psi(\cdot)\) is a cost function associated to investment. Smets and Wouters (2002) introduces an additional disturbance to the investment adjustment cost such as:

\[
K_{t+1} = (1-\delta) K_t + \left[ 1 - \Psi \left( \frac{V_t I_t}{I_{t-1}} \right) \right] I_t
\]

(5.6)

where \(V_t\) is assumed to follow an autorregressive process of order 1, \(\log V_t = \rho V \log V_{t-1} + \varepsilon^V_t\).

It is assumed that \(S_t = I_t\). The Lagrangian function associated to the household maximization problem can be defined as:

\[
\text{Max } \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \gamma \log(C_t) + (1-\gamma) \log(1-L_t) \right] \\ -\lambda_t (C_t + I_t - W_t L_t - R_t K_{t-1}) \\ -Q_t \left[ K_t - (1-\delta) K_{t-1} - \left[ 1 - \Psi \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \right] \right\}
\]

(5.7)

where \(Q_t\) is the Lagrange’s multiplier associated to the dynamics of capital stock. This multiplier, representing the shadow price of capital, is also known as the Tobin Q ratio and can be defined as the market value of the total installed capital over the replacement cost of that capital.
First order conditions for maximization are given by:

\[
\frac{\partial L}{\partial C} : \gamma \frac{1}{C_t} - \lambda_t = 0
\] (5.8)

\[
\frac{\partial L}{\partial L} : - (1 - \gamma) \frac{1}{1 - L_t} + \lambda_t W_t = 0
\] (5.9)

\[
\frac{\partial L}{\partial K} : \beta_t \left[ \lambda_t R_t + (1 - \delta) Q_t \right] - \beta^{t-1} Q_{t-1} = 0
\] (5.10)

\[
\frac{\partial L}{\partial I_t} : \beta_t \left[ Q_t - Q_t \Psi \left( \frac{I_t}{I_{t-1}} \right) - Q_t \Psi' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \lambda_t \right] + E_t \beta^{t+1} Q_{t+1} \Psi' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 = 0
\] (5.11)

We can define the Tobin's Q marginal ratio, named \( q_t \), as:

\[
q_t = \frac{Q_t}{\lambda_t}
\] (5.12)

that is, the ratio of the two Lagrange's multipliers. Therefore, we get that

\[
Q_t = q_t \lambda_t
\]

Using the FOC for the capital stock, we obtain:

\[
\frac{\lambda_{t-1}}{\lambda_t} Q_{t-1} = \beta \left[ \lambda_t R_t + (1 - \delta) Q_t \frac{\lambda_t}{\lambda_{t-1}} \right]
\]

or alternatively,

\[
q_{t-1} = \beta \frac{\lambda_t}{\lambda_{t-1}} [q_t (1 - \delta) + R_t]
\]

The above expression indicates that the value of current installed capital depends on its future expected value, taking into account the depreciation rate and the expected rate of return.

Moreover, operating in the first order condition for investment, we obtain:

\[
\beta_t \frac{\lambda_t}{\lambda_t} \left[ Q_t - Q_t \Psi \left( \frac{I_t}{I_{t-1}} \right) - Q_t \Psi' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \lambda_t \right]
\]

\[
= -E_t \beta^{t+1} Q_{t+1} \Psi' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2
\]

and substituting,

\[
\beta_t \left[ q_t - q_t \Psi \left( \frac{I_t}{I_{t-1}} \right) - q_t \Psi' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - 1 \right]
\]

\[
= -E_t \beta^{t+1} q_t \Psi' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2
\]
or

\[ q_t - q_t \Psi \left( \frac{I_t}{I_{t-1}} \right) - q_t \Psi' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} + E_t \beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \Psi' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 = 1 \]

Notice that if \( \Psi(\cdot) = 0 \), that is, there are no adjustment costs in investment, and then \( q_t = 1 \), that is the Tobin’s marginal Q should be equal to the replacement cost of installed capital in units of the final good.

By combining expressions (5.8) and (5.9) we obtain the condition that equates the marginal disutility of additional hours of work with the marginal return on additional hours:

\[ \frac{1 - \gamma}{\gamma} \frac{C_t}{1 - L_t} = W_t \]

Combining (5.8) and (5.10) we obtain the following equilibrium condition for the consumption path that equates the marginal rate of consumption with the rate of return of investment:

\[ q_{t-1} = \beta \frac{C_{t-1}}{C_t} [q_t (1 - \delta) + R_t] \]

The firms

The problem of firms is to find optimal values for the utilization of labor and capital. The production of final output \( Y \) requires the services of labor \( L \) and \( K \). The firms rent capital and employ labor in order to maximize profits at period \( t \), taking factor prices as given. The technology is given by a constant return to scale Cobb-Douglas production function,

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \] (5.13)

where \( A_t \) is a measure of total-factor, or sector-neutral, productivity and where \( 0 \leq \alpha \leq 1 \).

The static maximization problem for the firms is:

\[ \max_{(K_t, L_t)} \Pi_t = A_t K_t^\alpha L_t^{1-\alpha} - R_t K_t - W_t L_t \] (5.14)

The first order conditions for the firms profit maximization are given by

\[ \frac{\partial \Pi_t}{\partial K_t} : R_t - \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} = 0 \] (5.15)

\[ \frac{\partial \Pi_t}{\partial L_t} : W_t - (1 - \alpha) A_t K_t^\alpha L_t^{-\alpha} = 0 \] (5.16)
From the above first order conditions, equilibrium wage and rental rate of capital are given by:

\[ R_t = \alpha A_t K_t^{a-1} L_t^{1-a} \]  
\[ W_t = (1 - \alpha) A_t K_t^a L_t^{-a} \]

(5.17)  
(5.18)

**Equilibrium of the model**

For the equilibrium of the model, we first specify a particular functional form for the investment adjustment cost function. The literature offers a variety of different specifications. For instance, Christiano, Eichenbaum and Evans (2001) specify an adjustment cost function that satisfies the following properties \( \Psi(1) = \Psi'(1) = 0, \Psi''(1) > 0, \Psi_I(\cdot) = 1 \) and \( \Psi_{I^{-1}}(\cdot) = 0 \). Christoffel, Coenen and Warne (2007) use the following functional form:

\[ \Psi \left( \frac{I_t}{I_{t-1}} \right) = \frac{\psi}{2} \left( \frac{I_t}{I_{t-1}} - g_z \right)^2 \]

where \( \psi > 0 \) and \( g_z \) is the productivity growth rate in the long-run. Alternatively, Canzoneri, Cumby and Diba (2005) consider adjustment costs in capital, defining the following capital accumulation equation:

\[ K_t = (1 - \delta) K_{t-1} + I_t - \frac{\psi}{2} \left( \frac{I_t - \delta K_{t-1}}{K_{t-1}} \right)^2 \]

(5.20)

In our case, the investment adjustment cost function to be used is the following:

\[ \Psi \left( \frac{I_t}{I_{t-1}} \right) = \frac{\psi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \]

(5.21)

Thus, the equilibrium condition for investment can be written as:

\[ q_t - q_t \frac{\psi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - q_t \frac{I_t}{I_{t-1}} - \frac{I_t}{I_{t-1}} \]

\[ + \beta \frac{C_t}{C_{t+1}} q_{t+1} \psi \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 = 1 \]

**Equations of the model and calibration**

The competitive equilibrium of the model economy is given by a set of nine equations, driving the dynamics of the eight macroeconomic endogenous variables, \( Y_t, C_t, I_t, K_t, L_t, R_t, W_t, q_t \) plus the Total Factor Productivity, \( A_t \), which it is assumed to follows an autorregressive process of order 1.
This set of equations is the following:

\[(1 - \gamma) \frac{1}{1 - L_t} = \gamma \frac{1}{C_t} W_t \]  

(5.22)

\[q_{t-1} = \beta \frac{C_{t-1}}{C_t} [q_t (1 - \delta) + R_t] \]  

(5.23)

\[Y_t = G_t + I_t \]  

(5.24)

\[Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \]  

(5.25)

\[K_{t+1} = (1 - \delta) K_t + \left[1 - \frac{\psi}{2} \left(\frac{I_t}{I_{t-1}}\right)^2\right] I_t \]  

(5.26)

\[W_t = (1 - \alpha) A_t K_t^{\alpha} L_t^{-\alpha} \]  

(5.27)

\[R_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \]  

(5.28)

\[q_t - q_t \frac{\psi}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2 - q_t \psi \left(\frac{I_t}{I_{t-1}} - 1\right) \frac{I_t}{I_{t-1}} + \frac{\beta C_t}{C_{t+1}} q_{t+1} \psi \left(\frac{I_t}{I_{t-1}} - 1\right) \left(\frac{I_{t+1}}{I_t}\right)^2 = 1 \]  

(5.29)

\[\ln A_t = (1 - \rho_A) \ln A_t + \rho_A \ln A_{t-1} + \epsilon_t \]  

(5.30)

To calibrate the model economy, we need to assign values to the following parameters:

\[\Omega = \{\alpha, \beta, \gamma, \delta, \psi, \rho_A, \sigma_A\} \]

The only new additional parameter relative to the basic model is \(\psi\), which represents the intensity of adjustment costs in investment. Table 5.1 shows the calibrated values of the parameters. Since the literature uses different specifications for investment adjustment cost function, this leads to different calibrated values for the parameters representing the intensity of the adjustment costs. Smets and Wouters (2003) estimate a parameter of 5.9 for an adjustment cost function similar to the one used here. Christoffel et al. (2008) estimate a value of 5.8. Here, we will use a value of 6 as in the above works.

**Total Factor Productivity Shock**

This section studies how the presence of investment adjustment costs influences the effects of a positive shock in total factor productivity. Impulse-response functions for the variables of the model economy are plotted in Figure 5.1. The dynamic responses of the variables exhibit some notable
Table 5.1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital technological parameter</td>
<td>0.350</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.970</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Preference parameter</td>
<td>0.400</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.060</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Investment adjustment cost</td>
<td>6.000</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>TFP autoregressive parameter</td>
<td>0.950</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>TFP standard deviation</td>
<td>0.010</td>
</tr>
</tbody>
</table>

differences compared to the ones obtained from the DSGE model without adjustment costs in investment. First, as expected, we observe a different response of investment to the shock. Impulse-response of investment is now hump-shaped, implying a different transmission mechanism of the shock to capital stock and output. This response is explained by the existence of adjustment costs associated with investment, which reduces the change in the amount invested from one period to another. This response of investment increases the persistence in the capital stock accumulation process.

Another interesting result is the q-ratio response to the productivity shock. The positive productivity shock causes this ratio to rise above its steady state value, which by definition is 1. This means that it is profitable to invest, since in this case the rise in the market value of the firms is larger than the cost of the new capital. As the capital stock increases, the q-ratio decreases (given the decreasing marginal productivity of capital).

Conclusions

This chapter develops a DSGE model with adjustment costs in the investment process. Without investment adjustment costs, firms can adjust their capital stock to the optimal level instantaneously. Adjustment costs introduce an additional cost in the investment process as installation of new capital is not free, and hence, any difference between the optimal capital stock and the already installed capital stock could not be compensated in each period. This implies a different response of investment to shocks (investment is smoother) which translates into a higher persistence in the capital stock accumulation process. Investment adjustment costs have been introduced in the standard DSGE model as an important factor to describe investment dynamics and to explain some business cycle facts. Irreversibil-
Figure 5.1: TFP shock with investment adjustment costs

ity of capital stock, learning costs associated to the installation of new capital and labor adjustment costs are also important features to explaining capital and investment processes.

Appendix A: Dynare code

The Dynare code corresponding to the model developed in this chapter, named model5.mod, is the following:

// Model 5: Investment adjustment costs
// Dynare code
// File: model5.mod
// José L. Torres. University of Málaga (Spain)
// Endogenous variables
var Y, C, I, K, L, W, R, q, A;
// Exogenous variables
varexo e;
// Parameters
parameters alpha, beta, delta, gamma, psi, rho;
// Calibration of the parameters
alpha = 0.35;
beta = 0.97;
delta = 0.06;
gamma = 0.40;
psi = 2.00;
rho = 0.95;

// Equations of the model economy

model;
C=(gamma/(1-gamma))*(1-L)*W;
q=beta*(C/C(+1))*(q(+1)*(1-delta)+R(+1));
q-q*psi/2*((I/I(-1))-1)^2-q*psi*((I/I(-1))-1)*I/I(-1)
+beta*C/C(+1)*q(+1)*psi*((I(+1)/I)-1)*(I(+1)/I)^2=1;
Y = A*(K(-1)^alpha)*(L^(-1-alpha));
K = (1-delta)*K(-1)+(1-(psi/2*(I/I(-1)-1)^2))*I;
I = Y-C;
W = (1-alpha)*A*(K(-1)^alpha)*(L^(-1-alpa));
R = alpha*A*(K(-1)^alpha-1)*(L^(-1-alpha));
log(A) = rho*log(A(-1)) + e;
end;

// Initial values
initval;
Y = 1;
C = 0.8;
L = 0.3;
K = 3.5;
I = 0.2;
q = 1;
W = (1-alpha)*Y/L;
R = alpha*Y/K;
A = 1;
e = 0;
end;

// Steady state
steady;

// Blanchard-Kahn conditions
check;

// Perturbation analysis
shocks;
var e; stderr 0.01;
end;

// Stochastic simulation
stoch_simul;
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